Testable Consequences of Curved-Spacetime Renormalization

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Abstract

I consider certain renormalization effects in curved spacetime quantum field theory. In the very early universe these effects resemble those of a cosmological constant, while in the present universe they give rise to a significant finite renormalization of the gravitational constant. The relevant renormalization term and its relation to elementary particle masses was first found by Parker and Toms in 1985, as a consequence of the "new partially summed form" of the propagator in curved spacetime. The significance of the term is based on the contribution of massive particles to the vacuum. In the present universe, this renormalization term appears to account for a large part or even all of the Newtonian gravitational constant. This conjecture is testable because it relates the value of Newton's constant to the elementary particle masses.

I. INTRODUCTION

The observational determination of cosmological parameters appears to have reached a new level of accuracy. At the same time, there are longstanding unsettled questions such as the origin of dark-matter and large-scale structure, as well as important new questions.

Among these is the possibility of a significant non-zero cosmological constant implied by observations of type Ia supernovae by two independent groups. [1] Because of these new observations, I reconsidered a relatively large renormalization term that arises as a consequence of general relativity and quantum field theory in curved spacetime, [2] in the hope that it could explain the non-zero cosmological constant. This was a particularly appealing possibility because the relevant term arises through renormalization of the cosmological constant. Although this term does have significant and potentially observable consequences, and does act like a cosmological constant at very early times, it appears that at the present time its main effect is to produce a large finite renormalization of the gravitational constant. Although it does not seem capable of producing the acceleration of the expansion of the universe that is implied by the supernovae observations, it nevertheless has consequences that are testable.

In 1985, Parker and Toms [2] evaluated the gravitational part of the quantum corrections to the general relativistic effective action for a very general set of elementary particle quantum fields in an arbitrary curved spacetime. They showed that if there are particles of high mass contributing to the vacuum energy, as is predicted by elementary particle and string theories, then there would be a significant renormalization of the cosmological constant term in the Einstein-Hilbert action, and that the magnitude of the renormalization term would be of importance even in the present universe. Here, I consider more fully the role played by this renormalization term.

The relevant calculation in Ref. [2] is a direct evaluation of the gravitational part of the effective action that arises from vacuum fluctuations of quantum fields. The renormalization terms in the effective action that are of interest here come from non-perturbative (also called non-local) [3] contributions to the propagators of quantum fields in curved spacetime. The non-perturbative contribution comes from summing, for arbitrary dimensional spacetimes, an infinite series consisting of *all* terms that contain any factors of the scalar curvature R in the "proper-time" or Schwinger-DeWitt expansion of the propagator in curved spacetime [4,5]. The coefficients in the proper-time expansion contain successively higher powers of the

Riemann tensor, and its covariant derivatives and contractions. The “new partially summed form of the propagator” that results from summing all terms containing factors of R is manifestly generally covariant and contains a non-perturbative exponential factor involving the scalar curvature, multiplied by a proper-time series of terms having (in arbitrary dimension) no factors of R. When the non-perturbative exponential factor is expanded and the two series are multiplied one recovers the original expansion. The resulting terms that involve R are quite complicated, and are not known beyond a few orders. When the theorem stating that *all* of the R terms are summed in the new partially summed form of the propagator was first conjectured and proved [4,5], only the first three terms in the series expansion of the propagator had been calculated (because of the technical difficulty of calculating such complicated expressions). Since that time, several more terms have been calculated and shown to be consistent with the new form of the propagator. It is precisely because the exponential factor in the proper-time series for the propagator is extracted through the summation of the *complete* infinite series of highly complicated terms that involve R that its physical consequences must be regarded as realistic predictions of general relativity and quantum field theory. Although there may be other non-perturbative contributions to the propagator, this exponential one involving R is particularly significant because it gives rise to all the renormalization effects that involve R in the gravitational part of the effective action.

I first summarize the result derived in [2] for the renormalized cosmological constant. Then I discuss briefly its implications in the very early universe and at more length its implications in the present universe.

II. RENORMALIZED COSMOLOGICAL/GRAVITATIONAL CONSTANT

In the discussion of the cosmological constant and the Newtonian gravitational constant in Ref. [2], the particle content of the theory under consideration was quite general, as already noted. The part of the effective action relevant to our discussion was found to have

the form [see Eq.(3.54) of Ref. [2]],

$$\Gamma = \int dv_x (\Lambda_{\rm eff} + \kappa_{\rm eff} R + \cdots), \tag{1}$$

where the quantum corrections to $\kappa_{\rm eff}$ caused by particles of mass m were significant only at times when the curvature was of the order m^2 or larger. On the other hand, the quantum corrections to the term $\Lambda_{\rm eff}$ containing the cosmological constant, as noted in Ref. [2], is still of interest at the present time. The general form of $\Lambda_{\rm eff}$ is given (in units with $\hbar = c = 1$) in Eq.(4.5) of Ref. [2]:

$$\Lambda_{\rm eff} = \Lambda - \sum_i A_i N_i {m_i}^4 \ln[1 + B_i R/{m_i}^2]. \tag{2}$$

Here Λ is a constant energy-density which must be determined by observation. Classically, the value of the cosmological constant $\Lambda_{\rm c}$ is related to the value of the constant Λ in the action by [6] $\Lambda_{\rm c} = (8\pi G)\Lambda$ where G is the Newtonian gravitational constant. Here the sum is over the various species of particles of multiplicities N_i and masses m_i, and the dimensionless constants A_i and B_i are of order 1. The sum includes the contributions from particles of spin-0 and of spin-1/2. The contributions from particles of spin-1 are somewhat more complicated, but are sufficiently illustrated by the terms shown. For example, for spin-0 particles $A_i = (64\pi^2)^{-1}$, and $B_i = \xi_i - 1/6$, where ξ_i is a dimensionless coupling constant appearing in a term of the form $\xi_i R {\phi_i}^2$ in the Lagrangian of the scalar field ϕ_i. Since the fields of interest are highly massive, there is no conformal symmetry to favor any particular value of ξ. Therefore, it is natural to assume that $|B_i|$ is of order 1. The sign of the contributions to the sum in $\Lambda_{\rm eff}$ depends on the sign of B_i.

In the early universe, when R is of the order of the particle masses, the logarithmic terms in $\Lambda_{\rm eff}$ vary slowly and act much like a cosmological constant. Their effect in the early universe must be added to those of symmetry breaking scalar fields that may also contribute to a cosmological constant by being in the false vacuum. As the universe expands and R goes from larger to smaller than the particle masses, one must do a numerical integration to determine the effect of the logarithmic renormalization terms on the evolution. In addition,

variation of these terms with position and time will affect the growth of perturbations at early times.

Because of the factors of ${m_i}^4$, the main contribution to the sum in Eq.(2) comes from the largest masses, and except in the very early universe, the logs containing them can be expanded. For brevity, I will suppress the summation sign and denote by M and N the masses m_i and multiplicities N_i of any particle types that in recent times satisfy the condition, $|B_i R/{m_i}^2| \ll 1$. To good approximation, in recent times Eq.(2) becomes

$$\Lambda_{\rm eff} = \Lambda - ABNM^2R. \tag{3}$$

Then the effective action of Eq.(1) describing the recent universe becomes,

$$\Gamma = \int dv_x[\Lambda + (\kappa - ABNM^2)R + \cdots], \tag{4}$$

where terms other than the leading order in M^2 have been neglected in $\Lambda_{\rm eff}$ and $\kappa_{\rm eff}$, and κ is the constant contribution (if any) to $\kappa_{\rm eff}$. Thus, in recent times the main effect of the gravitational renormalization terms is to cause a large finite renormalization of the gravitational constant. This same renormalization of the gravitational constant also follows from the gravitational field equation corresponding [7] to Eq.(4.2) of Ref. [2]. The value of Newton's gravitational constant G in recent times (and in fact going back to times when the magnitude of R first became small with respect to M^2 for the most massive particles) satisfies

$$(16\pi G)^{-1} = (\kappa - \sum_i A_i B_i N_i M_i^2), \tag{5}$$

where the summation has been restored.

It is conceivable that independent limits on the value of the constant κ may come from considering the evolution of the very early universe. However, a plausible conjecture may be made at this time. If, as expected, there are massive particles having masses that are at the GUT scale or a significant fraction of the Planck scale, then the sum in Eq.(5) is sufficiently large that it may account for the full value of Newton's constant. If we assume

that this is the case, i.e., that κ is negligible compared to the sum, then we obtain a testable relation between the masses and multiplicities of the most massive particles and the value of Newton's constant G. Writing $G = M_{\rm Pl}{}^{-2}$, where $M_{\rm Pl} = 1.2 \times 10^{19}$ GeV, this relation is

$$(16\pi) \sum_i A_i B_i N_i (M_i / M_{\rm Pl})^2 = -1, \tag{6}$$

where the summation is over each type of particle that, in the present universe, satisfies the condition $|B_i R / M_i{}^2| \ll 1$. If the sum is dominated by N scalar bosons of mass M, then $A = (64\pi^2)^{-1}$, and if we assume that $B \approx -1$, then Eq.(6) would imply that

$$(4\pi)^{-1} N (M/M_{Pl})^2 \approx 1. \tag{7}$$

This appears to be a reasonable approximate relation in the light of current theories that predict multiplets of elementary particles having very large masses. With $B \approx -1$, the corresponding terms in the sum in Eq.(2) will give a positive contribution to the effective cosmological constant in the early universe, favoring early inflation.

In summary, the conjecture that in recent times the coefficient of the scalar curvature R in the effective action is dominated by the summation term in Eq.(5) gives Eq.(6), which can immediately be tested against theories of elementary particles and strings. At the level of quantum field theory in curved spacetime, this conjecture is a new embodiment of Sakharov's idea of induced gravity. [8,9] In a fully unified finite theory, it may be hoped that a relation of the type of Eq(6) will follow entirely from the theory, without the need for subsidiary assumptions such as the smallness of the constant κ relative to the sum in Eq.(5). Nevertheless, the relation of Eq.(6) between Newton's constant and the elementary particle masses is a fairly plausible and testable consequence of quantum field theory and general relativity.

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